

## Comparing the calculated coefficients of performance of a class of wind turbines that produce power between 330 kW and 7,500 kW

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**ABSTRACT:** The calculated coefficients of performance ( $C_p$ ) of wind turbines that produce power between 330 kW and 7,500 kW were compared using wind speeds that varied from 1 m/s to 25 m/s. The largest coefficients of performance were found to occur with wind speeds between 5 m/s and 10 m/s. Results show that each turbine is capable of producing coefficients of performance of, or near, 0.5; and the turbines studied proved to have essentially the same coefficients of performance, when exposed to similar wind speeds. Consequently, turbines with larger rotor diameters are not necessarily more efficient than those with smaller diameters, or vice versa. Therefore, when it comes to energy production, these data indicate that the advantage of turbines with larger rotor diameters is not in their efficiency. Rather, it is in the larger areas swept by their rotors, which expose them to more incoming wind energy.

### INTRODUCTION

The power curve of a wind turbine is defined as the steady power delivered by the turbine as a function of a steady wind speed between the cut-in and cut-out speeds. The cut-in wind speed is the minimum wind speed at which the turbine blades overcome friction and begin to rotate. The cut-out speed is the speed at which the turbine blades are brought to rest, typically by means of a control system, in order to avoid damage from high winds [1]. A sketch of an ideal power curve is shown in Figure 1.

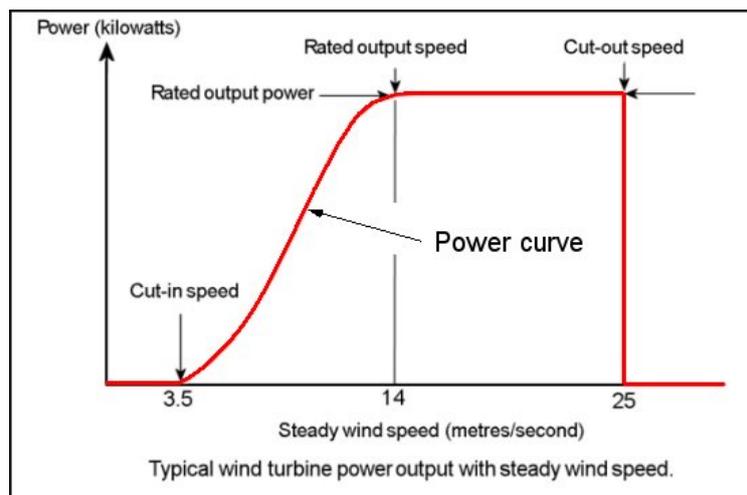


Figure 1: A sample plot of the power output of a wind turbine vs. steady wind speed [1].

Owners of wind turbines would like to know how much power their wind turbines can produce. Naturally, they are drawn to the maximum, or rated, output power shown on the power curve as provided by the manufacturer (Figure 1). The typical power curve provided by a manufacturer reflects the conditions in which it was developed. Unfortunately, the owner of a given wind turbine may not be able to duplicate the same exact conditions used by the manufacturer in developing the power curve [1]. This is because many factors affect the performance of a wind turbine [1-7]. These include environmental conditions such as wind patterns and winds change quite often; air pressure and temperature, which affect the mass density; turbulence intensity and variability of the wind; and wake effects and ice formation, see

for example [8-9]. Other factors that affect performance are related to the operation of the turbine, such as the condition of the turbine blades and of other turbine components; the suitability of the turbine to the site; how the movement of the turbine blades are controlled; the layout of the wind farm; where applicable, the schedule, regularity, thoroughness and quality of the maintenance; errors made during operation; and unexpected failures [2-10].

The deviations in measurements can be due to other factors, too. For example, current standards do not require that all parameters that affect power production be used in each test [11]. Thus, the parameters used by one manufacturer may not be the same as those used by another. Furthermore, there are wind turbines that do not have power curves that have been warranted by a manufacturer. Yet, owners of wind turbines need a way to estimate how much power their wind turbines can produce. In this article, data are presented that compare the calculated coefficients of performance of different wind turbines. These data came from turbines that produce power between 330 kW and 7,500 kW.

DESCRIPTION OF THE WIND TURBINES USED BY ENRON

Variability in the measured data on the performance of wind turbines has led ENERCON to calculate power yield forecasts for its wind turbines without using the power curves obtained by manufacturers of individual wind turbines. ENERCON calculates yields forecasts based upon certified power curve measurements obtained from accredited institutes [8]. ENERCON data were used in this article because they are considered to be both realistic and reliable in creating the expected energy that can be produced from the average speeds of the wind conditions at a given site. Table 1 summarises the characteristics of the turbines that were used by ENRON in their calculations [8].

Table 1: Technical specifications of ENERCON wind energy converters [8].

ENERCON Model	Rated Power (kW)	Rotor Diam. (m)	Tower Height (m)	Wind Zone	Turbine Wind Class	Swept Area (m <sup>2</sup> )	Rotational Speed (rpm)	Cut-out Speed (m/s)	Cut-in Speed (m/s)
E33	330	33.4	37-50	WZ II	IECI&II	876	18-45	28-34	2-3.5
E44	900	44	45-65	N/A	IECI	1,521	12-34	28-34	2-3.5
E48	800	48	50-76	WZIII	IECII	1,810	16-31	28-34	2-3.5
E53	800	52.9	60-75	WZ II	IECS	2,198	12-28.3	28-34	2-3.5
E70	2,300	71	57-113	WZIII	IECI&II	3,959	6-21.5	28-34	2-3.5
E82 E2	2,000	82	78-138	WZIII	IECII	5,281	6-18	28-34	2-3.5
E82 E2	2,300	82	78-138	WZIII	IECII	5,281	6-18	28-34	2-3.5
E82 E3	3,000	82	78-138	N/A	IECII	5,281	6-18.5	28-34	2-3.5
E101	3,000	101	99-135	WZIII	IECIII	8,012	4-14.5	28-34	2-3.5
E126	7,500	127	135	WZIII	IECI	12,668	5-11.7	28-34	2-3.5

In Table 1, each turbine has three blades. Each rotor is located upwind, is equipped with active pitch control and its rotation is clockwise [8]. The wind turbine classes and wind zones listed in Table 1 can be briefly explained as follows: during the conception, design and construction of a wind turbine, assumptions must be made regarding the wind climate within which the turbine will be operating. Wind turbine generator classes are just one of many factors that need to be considered. Three parameters are needed to determine the wind class: the average wind speed, turbulence intensity and extreme 50-year gust [12]. A region can be divided into wind zones, depending on the average speeds of the wind across that region. For example, wind zones in the United States are numbered I, II, III and IV. The regions that correspond to them are illustrated in Figure 2 [13].

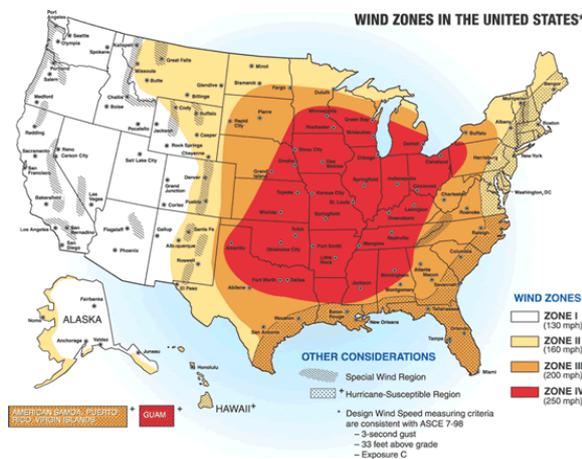


Figure 2: Wind zones in the USA.

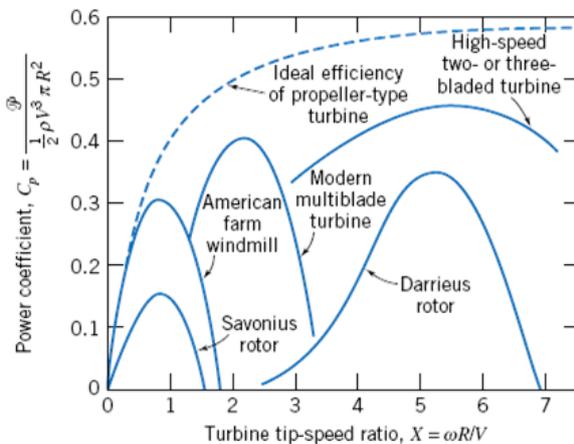


Figure 3: Power coefficient vs. tip-speed ratio [14].

## THE COEFFICIENT OF PERFORMANCE OF A WIND TURBINE

The coefficient of performance,  $C_p$ , also called the power coefficient of a wind turbine, is defined as the ratio of the power captured by the rotor of the wind turbine,  $P_R$ , divided by the total power available in the wind,  $P$ , just before it interacted with the rotor of the turbine. Using one-dimensional analysis, the coefficient of performance has been shown to have the expression shown below [4-7].

$$C_p = \frac{P_R}{P} = 4a(1 - a)^2$$

Where  $a$  represents the so-called interference factor. This factor is related to the reduction in the magnitude of the wind speed due to the presence of the wind turbine and is known to vary with the design of the wind turbine. It is also presumed to depend upon many variables including the speed, the angle of attack and the turbulence intensity of the wind, the geometry of the turbine blades, the material from which the blades are made, and the speed of rotation of the turbine rotor. Theoretically, the maximum value of the  $C_p$  is 0.593, and it is called the Betz limit. However, actual wind turbines have coefficients of performance that are much smaller than the Betz limit [15].

The calculated coefficients of performance of ten different wind turbines are shown in Table 2. It can be observed that the coefficient of performance of each wind turbine shown there increases as the wind speed increases to until a critical speed is reached. Performance decreases for speeds higher than this critical speed. These coefficients of performance were grouped by magnitude using increments of 0.1 and the resulting groupings were colour-coded in Table 2.

Using those colour codes as a guide, it can be seen that each turbine is capable of producing coefficients of performance of, or near to, 0.5. The largest coefficients of performance are found to occur between 5 m/s and 10 m/s. It can also be seen that turbines with larger rotor diameters are not necessarily more efficient than those with smaller diameters, or vice versa.

Table 2: Turbines used by ENRON and their calculated coefficients of performance as a function of wind speeds.

Model	E33	E44	E48	E53	E70	E82E2	E82E2	E82E3	E101	E126
Power KW	330	900	800	800	2,300	2,000	2,300	3,000	3,000	7,500
Wind (m/s)	Power Coeff.									
1	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0.19	0.1	0.12	0.12	0.12	0.076	0
3	0.35	0.16	0.17	0.39	0.27	0.29	0.29	0.29	0.279	0.263
4	0.4	0.34	0.35	0.44	0.36	0.4	0.4	0.4	0.376	0.352
5	0.45	0.43	0.43	0.46	0.42	0.43	0.43	0.43	0.421	0.423
6	0.47	0.48	0.46	0.48	0.46	0.46	0.46	0.46	0.452	0.453
7	0.5	0.49	0.47	0.49	0.48	0.48	0.48	0.48	0.469	0.47
8	0.5	0.5	0.48	0.49	0.5	0.49	0.49	0.49	0.478	0.478
9	0.5	0.5	0.5	0.49	0.5	0.5	0.5	0.5	0.478	0.477
10	0.47	0.5	0.5	0.48	0.5	0.49	0.49	0.49	0.477	0.483
11	0.41	0.48	0.45	0.42	0.49	0.42	0.44	0.44	0.439	0.47
12	0.35	0.44	0.39	0.34	0.45	0.35	0.38	0.39	0.358	0.429
13	0.28	0.39	0.32	0.27	0.39	0.29	0.32	0.35	0.283	0.381
14	0.23	0.33	0.27	0.22	0.34	0.23	0.26	0.3	0.227	0.329
15	0.18	0.28	0.22	0.18	0.28	0.19	0.22	0.26	0.184	0.281
16	0.15	0.24	0.18	0.15	0.23	0.15	0.18	0.22	0.152	0.236
17	0.13	0.2	0.15	0.12	0.19	0.13	0.15	0.19	0.127	0.199
18	0.11	0.17	0.13	0.1	0.16	0.11	0.12	0.16	0.107	0.168
19	0.09	0.14	0.11	0.09	0.14	0.09	0.11	0.14	0.091	0.142
20	0.08	0.12	0.09	0.08	0.12	0.08	0.09	0.12	0.078	0.122

21	0.07	0.11	0.08	0.06	0.1	0.07	0.08	0.1	0.067	0.105
22	0.06	0.09	0.07	0.06	0.09	0.06	0.07	0.09	0.058	0.092
23	0.05	0.08	0.06	0.05	0.08	0.05	0.06	0.08	0.051	0.08
24	0.05	0.07	0.05	0.04	0.07	0.05	0.05	0.07	0.045	0.071
25	0.04	0.06	0.05	0.04	0.06	0.04	0.05	0.06	0.04	0.063

In other words, the turbines studied have essentially the same coefficients of performance when exposed to similar wind speeds. Therefore, when it comes to energy production, these data indicate that the advantage of turbines with larger rotor diameters is primarily due to the fact they are exposed to more wind energy because of the larger swept areas created by their rotors.

Table 1 also shows that the rotor of each turbine is capable of performing rotations that fall within a definite range of speeds. The rotational speeds that the rotor of each turbine is capable of have been incorporated into the data shown in Table 2 and the results are presented in a different graphical format in Figures 4a and 4b.

Figure 4a illustrates the variation of the coefficients of performance with the tip-speed ratio, where the lowest rotational speed of the rotor was used. In Figure 4b, however, the tip-speed ratio was calculated using the highest rotation speed shown in Table 1. The tip-speed ratio is calculated by dividing the linear speed of the tip of the turbine blade by the speed of the wind. Therefore, it is a dimensionless ratio that indicates how much faster the tip of the blades moves compared with the speed of the air particles [16].

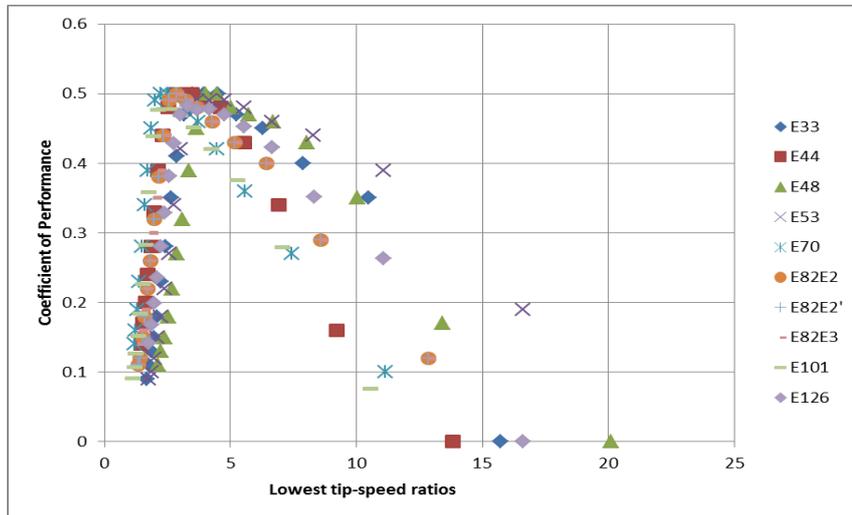


Figure 4a: Plots of the coefficients of performance of wind turbines vs. the lowest tip-speed ratios.

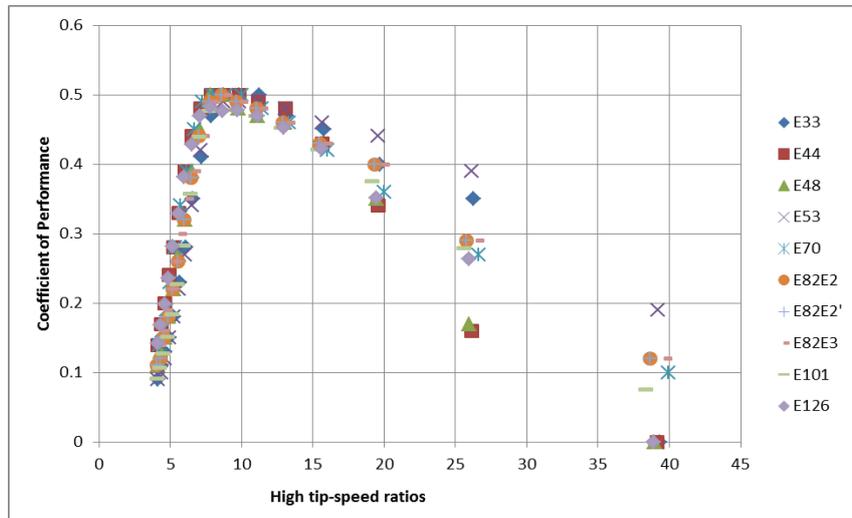


Figure 4b: Plots of the coefficients of performance of wind turbines vs. the highest tip-speed ratios.

## DISCUSSION AND CONCLUSIONS

Ten different wind turbines were considered. They were roughly similar in design but had different rotor diameters, different ranges of rotational speeds, and the magnitudes of their rated power ranged from 330 kW to 7,500 kW. Their coefficients of performance were calculated and compared using wind speeds that varied from 1 m/s to 25 m/s. Results are summarised below.

The effects of the wind speed: The largest coefficients of performance were found to occur with wind speeds between 5 m/s and 10 m/s, depending on the turbine used. Results showed that each turbine was capable of producing coefficients of performance of, or near, 0.5; and these turbines proved to have essentially the same coefficients of performance when exposed to similar wind speeds. Consequently, turbines with larger rotor diameters are not necessarily more efficient than those with smaller diameters, or vice versa. Therefore, when it comes to energy production, these data indicate that the advantage of turbines with larger rotor diameters is not in their efficiency. Rather, it is in the larger areas swept by their rotors, which expose them to more incoming wind energy to begin with.

The effects of the tip-speed ratio: The coefficients of performance were plotted against tip-speed ratios in Figures 4a and 4b. It can be observed that the coefficient of performance of each wind turbine increases as the tip-speed ratio increases until a critical tip-speed ratio is reached; performance decreases for higher tip-speed ratios. The largest coefficients of performance were found to occur with tip-speed ratios between 3 and 13, depending on the turbine used. It was also observed that for subcritical tip-speed ratios, the variation of the coefficients of performance across different turbines was smaller (that is less scatter); more scatter was observed in the data above the critical ratio and it increased with increasing ratios (Figure 4a). These data further suggest that, when turbines operate at their highest rotational speeds, the scatter in both the subcritical and supercritical regions are reduced considerably (Figure 4b). However, the range of tip-speed ratios that yield very good performance is wider in this case, suggesting that the reliability of performance is less sensitive to small changes in wind speeds.

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